

## WAVE EQUATION, MATHEMATICAL MODEL, AND REAL-LIFE EXAMPLES

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**Annotation:** This article discusses the mathematical model of the wave equation, its physical meaning, and its applications in various fields. The wave equation is a second-order partial differential equation that describes the propagation of energy in an elastic medium. The paper analyzes the d’Alembert solution, standing waves, and boundary conditions. Additionally, real-world applications of the wave equation—such as string vibrations, acoustic processes, electromagnetic waves, water surface waves, and seismic modeling—are described in detail. The article explains wave phenomena from a mathematical perspective and highlights their practical significance.

**Keywords:** wave equation, Dalembert solution, partial differential equations, acoustics, electromagnetic waves, seismic processes.

### Introduction

Waves are among the most fundamental and universal phenomena in nature, manifesting themselves in various forms of physical processes. The propagation of sound through air, oscillations on the surface of water, mechanical vibrations arising in elastic media, the electromagnetic nature of light, and seismic processes occurring in the Earth’s crust — all of these are closely related to the theory of waves. The wave equation serves as a universal mathematical model for such diverse processes. This equation constitutes one of the principal sections of the theory of partial differential equations and provides an accurate description of how energy propagates through space and time.

The universality of the wave equation lies in the fact that it retains the same mathematical form in different physical media. For example, although the vibration of a guitar string and the propagation of electromagnetic waves are entirely different physical phenomena, both can be described by the same mathematical equation. This makes the wave equation one of the fundamental laws of physics. In its general form, it is expressed as a second-order partial differential equation, and its solutions enable the study of the direction, shape, speed, and distribution of energy of waves.

The importance of the wave equation extends not only to theoretical physics and mathematics but also to various applied sciences. Today, modeling wave processes is essential in many fields such as acoustics, optics, radio communication,

telecommunication technologies, seismology, geophysics, civil engineering, and even biomedicine. For instance, seismic waves are used to study the structure of the Earth's layers, while radio waves ensure the functioning of global communication systems. Likewise, water-surface waves play a significant role in evaluating the efficiency of hydraulic and hydromechanical systems.

Therefore, this article provides a detailed analysis of the mathematical structure of the wave equation, its physical interpretation, solution methods, and areas of application in real life. Through this approach, the mathematical foundations of wave processes are comprehensively explained, offering insights into the general nature of complex wave phenomena encountered in various fields.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

Here,  $u(x,t)$  represents the displacement of the medium, and  $c$  is the wave propagation speed. This equation is directly related to the elasticity properties and density of the medium. The propagation speed is typically determined by the following formula:

$$c = \sqrt{\frac{T}{\rho}}$$

This formula is related to the tension and density of the string and explains the physical nature of wave propagation.

## 2. d'Alembert's Solution and Its Physical Interpretation

One of the most well-known analytical solutions of the wave equation was derived by the French mathematician d'Alembert:

$$c(x, t) = F(x - ct) + G(x + ct)$$

The physical interpretation of this solution is that, at any moment in time, the wave is a superposition of two independent waves propagating in opposite directions — to the right and to the left — with the same speed. If both ends of the string are fixed, stationary waves (standing waves), that is, oscillations with a constant shape, are formed as a result of the boundary conditions:

$$u(x, t) = A \sin\left(\frac{n\pi x}{L}\right) \cos(\omega t)$$

This form accurately describes the process of sound production in musical instruments.

## The Relationship Between the Wave Equation and Physical Models

### 1. Mechanical Waves

Vibrations in elastic strings, membranes, and spring-like media provide the simplest models of the wave equation. For example, the vibration of a guitar

string generates various harmonic modes depending on its length, tension, and density.

## 2. Acoustic Waves

The propagation of sound in air occurs through oscillations of pressure. For sound waves, the equation is expressed in three-dimensional form:

$$\frac{\partial^2 p}{\partial t^2} = c^2 \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \right)$$

This model is important in processes ranging from the design of acoustic rooms to the functioning of microphones and loudspeakers.

## 3. Electromagnetic Waves

The electromagnetic wave equation derived from Maxwell's equations forms the theoretical foundation of light, radio, Wi-Fi, and other communication technologies:

$$\frac{\partial^2 E}{\partial t^2} = c^2 \nabla^2 E$$

Here,  $E$  denotes the electric field intensity. Electromagnetic waves propagate in a vacuum at the highest possible speed — the speed of light.

## 4. Water Surface Waves

Water waves are generated under the influence of gravity and surface tension. Their mathematical model is expressed through complex differential equations. Such models are widely used in the design of hydraulic systems, reservoirs, and port infrastructure.

## 5. Seismic Waves

Waves generated by earthquakes in the Earth's crust are studied in geophysics using the three-dimensional elastic model of the wave equation. These processes make it possible to determine the internal structure of the Earth, layer densities, and the degree of seismic activity.

### Numerical Solutions and Modern Computational Methods

Analytical solutions are applicable only to idealized cases. Since real systems are complex, numerical methods such as:

- the Finite Difference Method (FDM),
- the Finite Element Method (FEM),
- spectral methods

are widely used. These methods allow for the evaluation of seismic safety of structures, the design of electromagnetic antennas, and the optimization of acoustic systems.

## Conclusion

The wave equation is a fundamental mathematical model that describes the most important processes in nature. It provides an understanding of the general laws governing various physical phenomena — from elastic strings to electromagnetic

fields. The universality of the wave equation has led to its application in numerous scientific and engineering fields. Through d'Alembert's solution, standing waves, and numerical modeling techniques, wave processes can be thoroughly analyzed, and their practical applications can be more precisely understood.

Modern technologies — radio communication, sensor systems, acoustic engineering, seismic monitoring, optical communication networks — are all built upon the mathematical foundations of the wave equation. Therefore, this equation is not only an essential component of theoretical mathematics but also of applied sciences and engineering.

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