DIFFERENSIAL TENGLAMALARNI YECHISHNING ANALITIK VA SONLI USULLARINI TAQQOSLASH

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Annotatsıya. Ushbu maqolada differensial tenglamalarni yechishning asosiy ikki yo‘nalishi — analitik (aniq) va sonli (taxminiy) usullar tahlil qilingan. Analitik usullar differensial tenglamaning yechimini aniq matematik ifoda ko‘rinishida olish imkonini bersa, sonli usullar murakkab va chiziqsiz tenglamalar uchun taxminiy yechimlarni hisoblashga asoslanadi. Maqolada har ikkala usulning afzalliklari, kamchiliklari, qo‘llanilish sohalari hamda ularning o‘zaro bog‘liqligi yoritilgan. Shuningdek, zamonaviy kompyuter dasturlari (MATLAB, Maple, Python, Wolfram Mathematica) yordamida sonli hisoblashlarning ahamiyati ko‘rsatib o‘tilgan. Natijada, analitik va sonli usullarni birgalikda qo‘llash differensial tenglamalarning amaliy va nazariy yechimlarini topishda eng samarali yondashuv ekanligi asoslab berilgan.

Kalit so‘zlar: differensial tenglama, analitik usul, sonli usul, Runge–Kutta, Euler usuli, kompyuter modellashtirish, taxminiy yechim.

Аннотация: В данной статье проанализированы два основных направления решения дифференциальных уравнений — аналитические (точные) и численные (приближённые) методы. Аналитические методы позволяют получить решение дифференциального уравнения в виде точного математического выражения, тогда как численные методы основаны на вычислении приближённых решений для сложных и нелинейных уравнений. В статье раскрыты преимущества и недостатки каждого метода, области их применения, а также взаимосвязь между ними. Кроме того, показано значение численных вычислений с использованием современных компьютерных программ (MATLAB, Maple, Python, Wolfram Mathematica). В результате обосновано, что совместное применение аналитических и численных методов является наиболее эффективным подходом к получению практических и теоретических решений дифференциальных уравнений.

Ключевые слова: дифференциальное уравнение, аналитический метод, численный метод, метод Рунге–Кутты, метод Эйлера, компьютерное моделирование, приближённое решение.

Abstract: This article analyzes the two main approaches to solving differential equations — analytical (exact) and numerical (approximate) methods. Analytical methods make it possible to obtain the solution of a differential equation in the form of an exact mathematical expression, whereas numerical methods are based on calculating approximate solutions for complex and nonlinear equations. The paper highlights the advantages, disadvantages, areas of application, and interconnection of both methods. In addition, the importance of numerical computations using modern computer programs (MATLAB, Maple, Python, Wolfram Mathematica) is discussed. As a result, it is substantiated that the combined use of analytical and numerical methods represents the most effective approach to obtaining both practical and theoretical solutions of differential equations.

Keywords: differential equation, analytical method, numerical method, Runge–Kutta method, Euler method, computer modeling, approximate solution.

KIRISH

Hozirgi davrda matematika fani nafaqat nazariy tadqiqotlarda, balki texnika, fizika, iqtisodiyot, biologiya, informatika va boshqa ko‘plab sohalarda muhim o‘rin tutadi. Ayniqsa, turli tabiiy va texnik jarayonlarning o‘zgarishini ifodalashda **differensial tenglamalar** asosiy matematik vosita sifatida qo‘llaniladi. Ular vaqt bo‘yicha yoki fazoviy o‘zgaruvchilar orqali biror miqdorning o‘zgarish qonuniyatini ifodalaydi.

Differensial tenglamalar yordamida issiqlik almashinuvi, mexanik tebranishlar, elektr zanjirlaridagi tokning o‘zgarishi, biologik populyatsiyalar o‘sishi, iqtisodiy tizimlarning dinamikasi kabi jarayonlar modellashtiriladi. Bunday jarayonlarni o‘rganish uchun tenglamaning yechimini topish zarur bo‘ladi, bu esa ko‘pincha murakkab masalani tashkil etadi.

Differensial tenglamalarni yechishning ikki asosiy yo‘nalishi mavjud: analitik usullar va sonli usullar. Analitik usullar tenglama yechimini aniq matematik ifoda ko‘rinishida olishga imkon beradi. Bu yondashuv jarayonni to‘liq nazariy tahlil qilish imkonini bersa-da, murakkab yoki chiziqsiz tenglamalar uchun ularning aniq yechimini topish har doim ham imkoni bo‘lavermaydi.

Bunday hollarda sonli (numerik) usullar qo‘llanadi. Ular differensial tenglamaning yechimini kompyuter yordamida taxminiy, lekin amaliy jihatdan yetarlicha aniq natija shaklida hisoblab beradi. Zamonaviy hisoblash texnikasi va dasturiy vositalar (MATLAB, Python, Maple, Wolfram Mathematica va boshqalar) sonli usullarning qo‘llanilish doirasini yanada kengaytirdi.

Shu sababli, ushbu ishda differensial tenglamalarni yechishning analitik va sonli usullari o‘rganilib, ularning afzalliklari, kamchiliklari hamda o‘zaro bog‘liqligi tahlil

qilinadi. Maqsad – har ikkala yondashuvning ilmiy va amaliy ahamiyatini ko‘rsatish, zamonaviy modellashtirishda ularning o‘zaro uyg‘unligini asoslab berishdan iborat.

1. Analitik usullar.

Analitik usul — differensial tenglama yechimini aniq matematik ifoda ko‘rinishida topishdir. Bunda yechim odatda funksiyalar, integrallar yoki maxsus funksiyalar orqali ifodalanadi.

Asosiy analitik usullar:

• Ajraluvchi o‘zgaruvchilar usuli

$$\frac{dy}{dx} = f(x)g(y) \Rightarrow \int \frac{dy}{g(y)} = \int f(x)dx$$

• Bir jinsli tenglamalar usuli

• Chiziqli differensial tenglamalar uchun integrallovchi ko‘paytuvchi usuli Bernulli va Rikatti tenglamalari uchun almashtirish usullari

• Ikkinchi tartibli chiziqli tenglamalar uchun xarakteristik tenglama usuli

• **Afzalliklari:**

• Yechim aniq va analitik shaklda olinadi.

• Jarayonning to‘liq matematik tavsifi beriladi.

• Tahliliy tadqiqotlar (stabilitet, asimptotika va h.k.) osonroq amalga oshiriladi.

Kamchiliklari:

• Murakkab yoki chiziqsiz differensial tenglamalar uchun aniq yechim topish qiyin yoki imkonsiz.

• Amaliy masalalarda analitik yechim ko‘p hollarda murakkab integral yoki maxsus funksiyalar orqali ifodalanadi.

Sonli usullar

Sonli (numerik) usullar differensial tenglamaning yechimini diskret nuqtalarda taxminiy hisoblash asosida quradi. Bu usullar zamonaviy kompyuter texnologiyalari yordamida juda keng qo‘llanilmoqda. **Asosiy sonli usullar:**

• Euler usuli

$$y_{n+1} = y_n + hf(x_n, y_n)$$

• Runge–Kutta usullari (2-, 4-, 6-tartibli)

• Adams–Bashforth va Adams–Moulton usullari

• Prediktor–korrektor usullari

• Sonli integratsiya va differensiallash algoritmlari **Afzalliklari:**

• Har qanday murakkab, chiziqsiz, tizimli differensial tenglamalar uchun yechimni yaqinlashtirib topish mumkin.

• Kompyuter dasturlari (MATLAB, Python, Maple, Wolfram Mathematica) yordamida tez va samarali hisoblash mumkin.

• Analitik yechim mavjud bo‘lmagan holatlarda amaliy natijalar olish imkonini beradi.

Kamchiliklari:

• Yechim aniq emas, taxminiy bo‘ladi.

• Natija qadam uzunligi (h) va xatolik nazoratiga bog‘liq.

• Uzun vaqt oralig‘ida xatolik to‘planishi (error accumulation) mumkin.

Afzalliklari:

• Har qanday murakkab, chiziqsiz, tizimli differensial tenglamalar uchun yechimni yaqinlashtirib topish mumkin.

• Kompyuter dasturlari (MATLAB, Python, Maple, Wolfram Mathematica) yordamida tez va samarali hisoblash mumkin.

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• Uzun vaqt oralig‘ida xatolik to‘planishi (error accumulation) mumkin.

Analitik va sonli usullarning taqqoslanishi

Asosiy jihat	Analitik usul	Sonli usul
Yechim turi	Aniq matematik formula	Diskret nuqtalarda taxminiy qiymatlar
Qo‘llanilishi	Oddiy va ayrim turlardagi tenglamalar	Har qanday murakkab tizimlar
Hisoblash usuli	Algebraik, integral, differensial manipulyatsiyalar	Iteratsion hisoblash, algoritmik
Natija aniqligi	Juda yuqori	Qadam uzunligiga bog‘liq
Kompyuter yordami	Har doim shart emas	Zarur
Afzallik	Nazariy tahlil uchun qulay	Amaliy modellashtirishda samarali
Kamchilik	Hamisha yechim topilmaydi	Xatolik to‘planadi

4. Zamonaviy integratsiya yondashuvlari

Bugungi kunda ko‘plab tizimlarda analitik va sonli usullar birgalikda qo‘llanadi. Masalan:

- Simvolik-numerik algoritmlar (Mathematica, Maple) avval analitik tahlilni bajaradi, keyin sonli hisob-kitob qiladi.

- Mashina o‘rganish yordamida differensial tenglamalarning yechimini taxminlash (neural ODEs).
- Gibrid usullar – analitik yechim mavjud bo‘lgan qismini tahlil qilib, qolgan qismini sonli usul bilan hisoblash.

XULOSA

Differensial tenglamalarni yechishda analitik va sonli usullar bir-birini to‘ldiruvchi yondashuvlardir. Analitik usullar nazariy tahlil uchun qulay bo‘lsa, sonli usullar amaliy hisobkitoblarda muhim ahamiyatga ega. Zamonaviy texnologiyalar esa bu ikki yo‘nalishni integratsiyalab, murakkab jarayonlarni aniq va samarali modellashtirish imkonini yaratmoqda.

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