ODDIY DIFFERENSIAL TENGLAMALARNI SONLI YECHISH, PARALLEL HISOBLASH USUL HAMDA ALGORITMLARINI QO'LLASH VA TADQIQ QILISH

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Anotatsiya: Ushbu maqolada oddiy differensial tenglamalarni sonli usullar yordamida yechishning asosiy yondashuvlari tahlil qilinadi. Euler usuli, Runge-Kutta usullari kabi keng tarqalgan algoritmlar ko‘rib chiqilib, ularning aniqligi, barqarorligi va qo‘llanish doiralari taqqoslanadi. Shuningdek, oddiy differensial tenglamalar uchun sonli yechimlarning amaliy misollari keltiriladi. Maqola ilmiy va amaliy sohalarda sonli usullarni qo‘llashga qiziqqan o‘quvchilar va mutaxassislar uchun foydali bo‘ladi.

Kalit so’zlar. Oddiy differensial tenglama, sonli usullar, Euler usuli, Runge-Kutta usullari, barqarorlik, aniqlik, sonli yechimlar.

Аннотация: В данной статье рассматриваются основные численные методы решения обыкновенных дифференциальных уравнений. Анализируются широко используемые алгоритмы, такие как метод Эйлера и методы Рунге-Кутты, с точки зрения точности, устойчивости и областей применения. Также приводятся практические примеры численных решений ОДУ. Статья будет полезна студентам и специалистам, интересующимся применением численных методов в научных и прикл адных задачах.

Ключевые слова. Обыкновенное, дифференциальное уравнение, численные методы, метод Эйлера, методы Рунге-Кутты, устойчивость, точность, численное решение

Annotation: This article analyzes the main numerical methods for solving ordinary differential equations (ODEs). Widely used algorithms such as the Euler method and Runge-Kutta methods are examined in terms of accuracy, stability, and applicability. Practical examples of numerical solutions for ODEs are also provided. The article is intended to be useful for students and professionals interested in applying numerical methods in scientific and practical fields.

Keywords: ordinary differential equation, numerical methods, Euler method, RungeKutta methods, stability, accuracy, numerical solution

Kirish

Oddiy differensial tenglamalar (ODT) ko‘plab tabiiy va texnik jarayonlarni tavsiflashda keng qo‘llaniladi. Ularning aniq yechimlarini topish har doim ham mumkin emas, ayniqsa, murakkab yoki chiziqli bo‘lmagan tenglamalar uchun. Shu

sababli, oddiy differensial tenglamalarni sonli usullar yordamida yechish dolzarb masalalardan biri hisoblanadi. Sonli usullar – bu matematik modelni kompyuter orqali taxminiy yechimlarini topish uchun ishlatiladigan algoritmlar majmui.

Eyler usuli va Runge-Kutta usullari kabi asosiy sonli yondashuvlar oddiy differensial tenglamalarning yechimlarini aniqlashda ko‘plab ilmiy va muhandislik masalalarida muvaffaqiyatli qo‘llaniladi. Ushbu maqolada ushbu sonli usullarning asosiy printsiplari, afzalliklari va kamchiliklari ko‘rib chiqiladi hamda ularning samaradorligi misollar yordamida tahlil qilinadi.

Eyler usuli – bu differensial tenglamaning yechimini bosqichma-bosqich yaqinlashish orqali taxminan topishga imkon beruvchi eng sodda sonli metodlardan biridir. U quyidagi umumiy formula asosida ishlaydi:

$$y_{n+1} = y_n + h \cdot f(x_n, y_n)$$ **Misol:**

$\frac{dy}{dx} = x + y, \quad y(0) = 1$

Eyler formulasi quyidagicha ifoalanadi:

$y_{n+1} = y_n + h \cdot f(x_n, y_n)$

Boshlang‘ich qiymatlar:

$$x_0 = 0, \quad y_0 = 1$$

Hisoblash jadvali:

n	x_n	y_n	$f(x_n, y_n) = x_n + y_n$	$y_{n+1} = y_n + h \cdot f(x_n, y_n)$
0	0.1	1.0000	1.0000	1.0000+0.1×.0000=1.1 000
1	0.2	1.1000	1.2000	1.1000+0.1× 1.2000=1.2 200
2	0.3	1.2200	1.4200	1.2200+0.1× 1.4200=1.3 620
3	0.4	1.3620	1.6620	1.3620+0.1× 1.6620=1.5 282
4	0.5	1.5282	-	-

Yakuniy natija:

$y(0.4) \approx 1.5282$

Yana bir misolni C# dasturida ko‘ramiz:

$\frac{dy}{dx} = x + y, \quad y(0) = 1, \quad h = 0.1, \quad x \in [0,0.5]$

C# kodi:

using System;

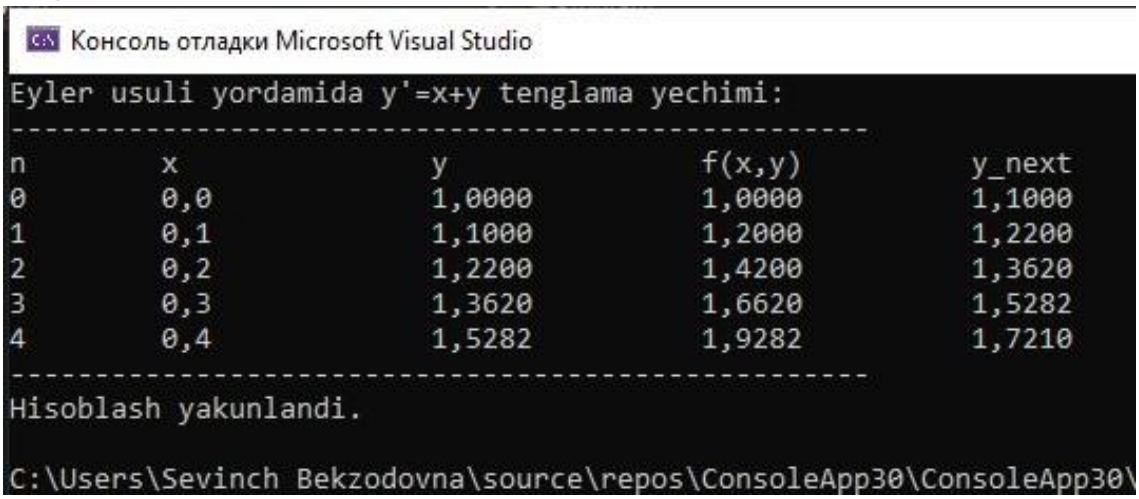
```
class Program
{ static void Main()
   { // Boshlang'ich shartlar        double x = 0.0;
double y = 1.0;
double h = 0.1;
int steps = 5;
 Console.WriteLine("Eyler usuli yordamida y'=x+y tenglama yechimi:");
       Console.WriteLine("-------------------------------------------------");
       Console.WriteLine("n\t x\t\t y\t\t f(x,y)\t\t y_next");

       for (int n = 0; n < steps; n++)
       { double f = x + y;
          double y_next = y + h * f;

          Console.WriteLine($"{n}\t {x:F1}\t\t {y:F4}\t\t {f:F4}\t\t {y_next:F4}");

          // Keyingi qadam           y = y_next;
x += h;}

       Console.WriteLine("-------------------------------------------------");
Console.WriteLine("Hisoblash yakunlandi.");  }
} Natija:
```



Xulosa:

Oddiy differensial tenglamalarni sonli usullar yordamida yechish bugungi kunda ilm-fan va muhandislik sohalarida muhim ahamiyatga ega. Ayniqsa, ularning analitik yechimi mavjud bo‘lmagan yoki murakkab bo‘lgan hollarda sonli yondashuvlar yagona amaliy vosita hisoblanadi.

Eyler usuli — eng sodda va tushunarli sonli metod bo‘lib, u orqali differensial tenglamaning yechimi bosqichma-bosqich taxminiy hisoblab chiqiladi. Bu usulda har

bir yangi qiymat avvalgi qiymatga hosilaning ko‘paytmasini qo‘shish orqali olinadi. Eyler usuli orqali olingan misol natijalari jadvalda aniq ko‘rsatilgan bo‘lib, bu metodning ishlash tamoyili amalda qanday ekanligini yaqqol tasvirlaydi.

Shuningdek, Eyler usulining asosiy afzalligi — uning soddaligi va hisoblashdagi tezligi bo‘lsa, kamchiligi — bu aniqlikning past darajada bo‘lishi va xatoliklarning bosqichmabosqich to‘planishidir. Shuning uchun u odatda boshlang‘ich taxminlar yoki oddiy modellarda qo‘llaniladi, murakkab vaziyatlarda esa undan aniqroq metodlar — masalan, Runge-Kutta usullari — afzal ko‘riladi.

Umuman olganda, Eyler usuli differensial tenglamalarni tushunish va sonli yechimlarga kirishish uchun qulay asos bo‘lib xizmat qiladi. Uni amaliy masalalarda qo‘llash orqali dastlabki natijalarni tez va oson olish mumkin, bu esa keyinchalik murakkabroq usullarga o‘tishda mustahkam poydevor bo‘ladi.

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